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DBW-003-1012008

Seat No. ______

B. Sc. (Sem. II) (CBCS) Examination

July - 2022

Mathematics : Paper - II (A) (Theory)

(Geometry, Calculus & Matrix Algebra)

(Old Course)

Faculty Code : 003

Subject Code : 1012008

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Attempt all questions.

(2) All questions carry equal marks.

1 (a) Answer the following questions briefly **4**

(1) Write Standard form of Sphere

(2) Define Cylinder

(3) Write the equation of cylinder whose axis is parallel to Y-axis and radius r.

(4) Find the centre and radius of the sphere

$$|\overline{r}|^2 + \overline{r} \cdot (-8, 6, -10) + 1 = 0.$$

(b) Attempt any **one** out of two : **2**

(1) Find the equations of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 3$ and the point $(1, 1, 1)$.

(2) Find the two tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 6y - 4z + 5 = 0$ which are parallel to the plane $2x - y + 2z = 0$.

(c) Attempt any **one** out of **two** : **3**

(1) Find equation of right circular cylinder with radius 4 and axis $x = 2y = -z$.

(2) Find the equation of the sphere passing through the points $O(0, 0, 0)$, $A(-a, b, c)$, $B(a, -b, c)$ and $C(a, b, -c)$

(d) Attempt any **one** out of two : **5**

(1) Derive the equation of right circular cylinder with axis $\frac{x-\alpha}{i} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ and radius r.

(2) A sphere of constant radius k passes through the origin and meets co-ordinate axis in A, B, C. Prove that the centroid of ΔABC lies on the sphere $9(x^2 + y^2 + z^2) = 4k^2$.

2 (a) Answer the following questions briefly : **4**

(1) Define isolated point of a set

(2) Evaluate $\lim_{x\to1}\left\{\lim_{y\to2} xy^2\right\}$

(3) Write formula of partial derivative $\frac{\partial f}{\partial y}$

(4) If $u = \log\left(x^2 + y^2\right)$ then find $\frac{\partial u}{\partial y}$.

(b) Attempt any **one** out of two : **2**

(1) If $u=\sin\left(\frac{x^2+y^2}{xy}\right)$ then prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0.$$

(2) Using definition prove that $\lim\limits_{(x,\, y)\to(1,\, 2)} xy=2$.

(c) Attempt any **one** out of two : **3**

(1) If u = log(tan$x$ + tan$y$) then prove that

$$\sin 2x\frac{\partial u}{\partial x}+\sin 2y\frac{\partial u}{\partial y}=2.$$

(2) If $W=\log\left(\frac{x^2+y^2}{xy}\right)$ then prove that

$$X\frac{\partial W}{\partial x}+Y\frac{\partial W}{\partial y}=0.$$

(d) Attempt any **one** out of two : **5**

(1) If $f(x,y)=\sin^{-1}\left(\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$ then show that

$$\frac{f_x}{f_y}=\frac{-y}{x}.$$

(2) State and prove Euler's theorem for homogeneous function of two variables.

3 (a) Answer the following questions briefly : **4**

(1) Define extreme point.

(2) Find the critical point of $f(x, y) = x^2 + 2y^2 - x$

(3) If $x = r\cos\theta$ and $y = r\sin\theta$ then find $\frac{\partial(r, \theta)}{\partial(x, y)}$.

(4) Define local maxima.

(b) Attempt any **one** out of two : **2**

(1) Find minimum value of

$$f(x, y) = x^2 + 2y^2 - 2xy - 2x + 2y + 1$$

(2) If $f(x, y) = x^2 y - 3y$ then find approximate value of f(5.12, 6.85).

(c) Attempt any **one** out of two : **3**

(1) If $u = \frac{x + y}{1 - xy}, v = \tan^{-1} x + \tan^{-1} y$ then show that

$$\frac{\partial(u, v)}{\partial(x, y)} = 0.$$

(2) If $u = x + y + z,\ uv = y + z,\ uvw = z$ then show that

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v.$$

(d) Attempt any **one** out of two : **5**

(1) If $u = \frac{yz}{x}, v = \frac{zx}{y}$ and $w = \frac{xy}{z}$ then show that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4.$$

(2) Expand $e^x \cos y$ in power of x and y.

4 (a) Answer the following questions briefly : **4**

(1) Define Diagonal matrix

(2) Give an example of 3×3 Symmetric matrix

(3) Define Involutary matrix

(4) Define Upper Triangular matrix

(b) Attempt any **one** out of two : **2**

(1) Show that there exists unique inverse matrix for any non singular matrix.

(2) If $A = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ then find $A + B$.

(c) Attempt any **one** out of two : **3**

(1) For non singular matrices A and B, prove that $(AB)^{-1} = B^{-1}A^{-1}$.

(2) Prove that $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$ is idempotent matrix.

(d) Attempt any **one** out of two : **5**

(1) Find the Rank of the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$.

(2) Prove that every square matrix can be expressed uniquely as the sum of a symmetric and skew symmetric matrix.

5 (a) Answer the following questions briefly : **4**

(1) Define consistent of equations.

(2) Find the characteristic equation of $\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$.

(3) Define characteristic polynomial of a matrix.

(4) Define eigen value.

(b) Attempt any **one** out of two : **2**

(1) Show that the matrices A and A^T have same eigen values.

(2) For matrix $\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$, find the eigen values.

(c) Attempt any **one** out of two : **3**

(1) Show that equation $AX = O$ has a non zero solution iff A is singular matrix.

(2) Solve : $x + y + z = 9$,

$2x + 5y + 7z = 52$,

$2x + y - z = 0$

(d) Attempt any **one** out of two : **5**

(1) Find eigen value and eigen vectors of matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

(2) State and prove Cayley-Hamilton Theorem.
